REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the revised indexing system printed in Volume 28, Number 128, October 1974, pages 1191–1194.

1[2.30, 7.50].–T. S. Chihara, *An Introduction to Orthogonal Polynomials*, Gordon and Breach Science Publishers, New York, London, Paris, 1978, xii + 249 pp., $23\frac{1}{2}$ cm. Price \$39.50.

Written at the beginning graduate level, this text covers selected aspects of orthogonal polynomials, the emphasis being on spectral properties that can be derived directly from the recurrence formula. Of the six chapters, the first four develop the subject in an orderly deductive fashion, while the last two essentially report on a large number of assorted facts without proofs.

Orthogonal polynomials are defined with respect to a linear functional $\mathcal{L}$ acting on the vector space of all polynomials. Thus, $\{P_n(x)\}_{n=0}^{\infty}$ is a sequence of orthogonal polynomials if each P_n has exact degree n, $\mathcal{L}(P_nP_m) = 0$ whenever $n \neq m$, and $\mathcal{L}(P_n^2) \neq 0$ for all $n \geqslant 0$. The functional is positive-definite if $\mathcal{L}(\pi) > 0$ for every polynomial $\pi \not\equiv 0$ which is nonnegative on the real line. Chapter I develops elementary properties of orthogonal polynomials, including existence criteria, the recurrence formula, basic facts on zeros, Gaussian quadrature, and kernel polynomials. Chapter II begins with the characterization of positive-definite functionals $\mathcal{L}$ in terms of the Stieltjes integral representation $\mathcal{L}(\pi) = \int_{-\infty}^{\infty} \pi(x)\, d\psi(x)$, where ψ is bounded nondecreasing and has an infinite spectrum (i.e., infinitely many points of increase). $\mathcal{L}$ is called determined, if ψ is essentially unique. While the author proves that functionals having a bounded supporting set are determined, he considers the general theory of determinacy as being outside the scope of this text. Likewise, there is only a brief introduction into the moment problems of Stieltjes and Hamburger. Chapter III, largely preparatory for the material in Chapter IV, deals with elementary properties of continued fractions and the connection between continued fractions and orthogonal polynomials. It also contains a rather detailed treatment of H. S. Wall's theory of chain sequences, i.e. numerical sequences $\{a_n\}_{n=1}^{\infty}$ in which each a_n, $n \geqslant 1$, admits a representation $a_n = (1 - g_{n-1})g_n$ with $0 \leqslant g_0 < 1$, $0 < g_n < 1$. The general theme of Chapter IV is the problem of extracting properties of orthogonal polynomials from the behavior of the coefficients in the recurrence formula. The properties in question concern boundedness and unboundedness of the true interval of orthogonality and structural properties of the spectrum of the underlying distribution ψ. The chapter culminates in Krein's theorem characterizing the case in which the spectrum is a bounded set with finitely many limit points.

Chapter V begins with a quick introduction into the "classical orthogonal polynomials" and their formal properties. Several unifying principles are then

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discussed of characterizing classical orthogonal polynomials in terms of differential equations, differentiation properties, or Rodrigues type formulas. Suitably generalized, these principles yield more general classes of orthogonal polynomials, including those of Hahn and Meixner. A large number of specific orthogonal polynomials, often with unusual, but interesting (discrete and continuous) weight distributions, are reviewed in Chapter VI. The book concludes with historical notes and an appendix. The appendix contains a table of recurrence formulae for all explicitly known (monic) positive-definite systems of orthogonal polynomials, treated or mentioned in the book.

While the text proper (Chapters I–IV) rarely gets down to specific cases, one finds many examples in the exercises. These, therefore, extend and illustrate the text in an essential way.

The author has deliberately chosen to omit many important subject areas related to orthogonal polynomials. Among these are (i) inequalities and asymptotic properties of orthogonal polynomials and their zeros, and of Christoffel numbers and Christoffel functions, (ii) integral representations, (iii) the deeper aspects of the moment problem, in particular, questions of determinacy and their interplay with closure properties and approximation theory, (iv) expansion theorems and summability theory, (v) the theory of polynomials orthogonal on the circle and on arbitrary curves, (vi) convergence of interpolation and quadrature processes based on the zeros of orthogonal polynomials. The rationale for these omissions, in the author's own (modest) words, is "... the fact that existing books contain better treatments of these topics than we could provide anyway." The book by Szegö [6] (and the appendix by Geronimus [4] to the Russian translation of the 1959 edition of Szegö's book), as well as the book by Freud [3] (and the recent memoir of Nevai [5]), indeed cover many of these topics in great depth. For questions of positivity that arise in connection with sums of orthogonal polynomials, expansion coefficients for one system of polynomials in terms of another, and integrals of orthogonal polynomials, as well as for addition theorems, the best source is the set of lectures by Askey [1]. Additional topics not covered in the book are applications of orthogonal polynomials in the physical and social sciences, computational considerations, and orthogonal polynomials in several variables.

The limited material covered, however, is presented in a well-organized and pleasing manner. A great deal of information, in part not previously available in book form, is packed into the relatively short span of 250 pages.

With regard to the history of orthogonal polynomials, the reviewer takes this opportunity to draw attention to an important paper of Christoffel [2] which has been consistently overlooked in all books on the subject, including the one under review. Christoffel appears to be one of the first, if not the first, to introduce orthogonal polynomials with respect to an arbitrary weight function (on a finite interval) and to initiate their systematic study.

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1. Richard Askey, *Orthogonal Polynomials and Special Functions*, Regional Conference Series in Appl. Math., No. 21, SIAM, Philadelphia, Pa., 1975.

2. E. B. Christoffel, "Sur une classe particulière de fonctions entières et de fractions continues," *Ann. Mat. Pura Appl.* (2), v. 8, 1877, pp. 1–10. [Also in: Ges. Math. Abhandlungen II, pp. 42–50.]

3. GÉZA FREUD, *Orthogonale Polynome*, Birkhäuser, Basel, 1969; English transl., Pergamon Press, New York, 1971.

4. JA. L. GERONIMUS & G. SZEGÖ, *Orthogonal Polynomials*, Amer. Math. Soc. Transl. (2), vol. 108, Amer. Math. Soc., Providence, R. I., 1977, pp. 37–130.

5. PAUL G. NEVAI, *Orthogonal Polynomials*, Mem. Amer. Math. Soc., Vol. 18, No. 213, Amer. Math. Soc., Providence, R. I., 1979.

6. GABOR SZEGÖ, *Orthogonal Polynomials*, 4th ed., Amer. Math. Soc. Colloq. Publ., Vol. 23, Amer. Math. Soc., Providence, R. I., 1975.

2[9.00].–PAULO RIBENBOIM, 13 *Lectures on Fermat's Last Theorem*, Springer-Verlag, New York, xi + 302 pp., 24 cm. Price $24.00.

This book will surely become one of the classics on Fermat's Last Theorem. In a very readable style, the author summarizes most of the important work relating to FLT and tries to give the main ideas that go into the proofs. The research has been rather thorough, and each chapter concludes with a long list of references. Starting with the early work on degrees up to seven, and also the results obtained by "elementary" methods, the author then proceeds to Kummer's work. He then treats more recent work, for example that of Wieferich, Mirimanoff, Vandiver, and Krasner. Next, the reader is treated to a discussion of applications of class field theory, linear forms in logarithms, elliptic curves, and congruences. Also included is a discussion of topics that have appeared in this journal, such as the tables of W. Johnson and S. Wagstaff and recent conjectures concerning the distribution of irregular primes and of the index of irregularity. The book concludes with a sometimes light-hearted treatment of variations of FLT: polynomials, differential equations, nonassociative arithmetics, etc. Because of a lack of space, and to enhance readability, proofs are often omitted or only sketched. But the interested reader can always consult the references, or wait for the promised second, more technical volume to be published. Most of the text should be accessible to a mathematician with an undergraduate course in number theory, if certain sections involving algebraic number theory are omitted. Though writing on a subject notorious for its errors, the author seems to be fairly accurate. However, we note two minor mistakes: on page 82 and 98 the words "positive real unit" should be replaced by "real unit" since positivity will vary with the embedding into the reals; on page 208 the formula for the genus should have a 4 instead of a 5.

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3[10.35].–BERNARD CARRÉ, *Graphs and Networks*, Clarendon Press, Oxford, 1979, x + 277 pp., 23cm. Price $36.50 (cloth), $19.50 (ppr.).

This book is a rather unusual entry into the literature on graphs and networks. Its motivation comes from operations research and computer science; thus, its applications include, for instance, critical path analysis, dynamic programming and assigning memory space when compiling a computer program. Its viewpoint is algorithmic and algebraic.